Solution of Midterm Exam

Problem 1

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Objective function :

As , no optimal u. u has been selected between 1 and -1.

The switch number is determined by the eigenvalues of the state equation.

We can also calculate the eigenvalues of the adjoint equation as . They are complex. Then the number the switches will depend on the initial condition.

In homework 2, the eigenvalues of the state equations are real number, then for nth order system, the maximum number of switches is n-1. In that problem, n=2, so there’s only 1 switch.

Problem 2

Apply z-transform:

As , , ,

We can have , , ,

Then we can have

,

(results near this value is also OK).

Problem 3

The observable canonical realization of this system is

( [http://en.wikipedia.org/wiki/Realization\_(systems)](http://en.wikipedia.org/wiki/Realization_%28systems%29) )

Here, , ,

Then the discrete form of this system is

Here,

As

Assume

Then apply z transform to

We have

From

We have

Make , then

Then we can have

Problem 4

State equation:

Adjoint equation:

The eigenvalues of is , then

As the eigenvalues of and are then same, then we have .

Then

So

The eigenvalues of adjoint equation is the negative of those of the state equations.

This will cause stability problem when we try to integrate the two equations in the same direction, because one of them must be unstable.

Problem 5

Riccati ODE:

Here, and are symmetric.

Take transpose on both sides of this equation, then we have

and are the solutions of the same equation, then it’s very easy to have

So is symmetric.

Problem 6

In continuous time domain, the optimal control is

In order to find the minimum value of the objective function, .

In order to make meaningful, should not be 0. Then must be positive definite.

In discrete time domain, the optimal control is

Then R is not required to be positive defined. It can be 0.

Problem 7